Complex modulus modeling of asphalt concrete mixes using the Non-Smooth Contact Dynamics method

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Abstract

Asphalt mixtures are complex multiphase materials showing a viscoelastic behavior. To assess the mechanical response of these materials, a typical practice is to perform a complex modulus test. An alternative to laboratory characterization is to simulate numerically this test by means of a discrete approach. These approaches are able to reproduce the mechanical performances of asphalt mixtures, but are still time-consuming. In this paper, the complex modulus test is reproduced numerically for a viscoelastic granular material by means of 3D Non-Smooth Contact Dynamics simulations. A viscoelastic phase surrounding the rigid particles is simulated by a contact model acting between them. This contact law was implemented in the LMGC90 software, based on the Burger’s model. The developed contact model handles larger time step lengths to reduce the computational time. Experimental and numerical testing campaigns were conducted for the complex modulus test on trapezoidal samples in a 2PB configuration. The numerical model was able to reproduce the mechanical performances obtained during experimental tests, regarding the material properties such as the complex modulus norm and the associate phase angle. The proposed model can be used to simulate the mechanical response of road structures under traffic loading concerning rutting, crack propagation and fatigue damage.

Keywords: Complex modulus test, asphalt concrete modeling, NSCD method, Burger’s model

1. Introduction

Geomaterials are widely used as filling materials in construction foundations, earthworks and transport infrastructures such as railway ballast and asphalt concrete. The latter is a complex multiphase mixture comprising mastic of bitumen and filler, graded mineral aggregates and air. This mixture links the granular phase, composed of crushed aggregates, with bitumen to provide cohesion to the mixture. This material has a typical viscoelastic behavior, where the mixture properties are function of strain rate and temperature. In these materials, the macroscopic behavior is strongly linked to the micro-scale behavior. Hence, to make accurate predictions regarding pavement performances, a clear understanding of the mechanical behavior of asphalt mixtures at the particle-scale is needed.

Complex modulus test allows quantifying the viscoelastic properties of asphalt mixtures. Among these properties, the most important are the norm of the complex modulus $|E^*|$ together with the associate phase angle $\phi$. These properties are used as inputs to calculate the asphalt pavement response under traffic loading and to design the thickness of asphalt concrete pavement layers. Laboratory tests are able to identify the macro-scale response of asphalt mixtures under cyclic loading, but this approach struggles to obtain a micromechanical insight of these materials. Numerical simulations using a continuum mechanics approach such as Finite Element Method (FEM) have been successfully used to describe the overall behavior of viscoelastic materials [1, 2, 3]. Nevertheless, these methods show the same difficulties as experimental approach regarding the identification of microscale properties.

A workaround for this issue is to use a discrete approach, which has been widely used to model the behavior of granular materials. This method allows simulating the interaction of a collection of rigid or deformable bodies in contact. Over the past two decades, the discrete element method (DEM) has been used in several studies to model the mechanical behavior of asphalt mixtures. This approach takes into account the granular nature of aggregates and the mastic behavior applying viscoelastic laws at particle contact [4, 5, 6, 7, 8]. These studies have modeled the asphalt using simple spherical particles, irregular particles created with clumps of spheres or by generating samples using imaging techniques [9, 10, 11, 12].

Despite the accurate micromechanical behavior predictions of asphalt mixtures using classic DEM approach, this numerical method is still very time-consuming. This issue is due in part to the length of the time step $dt$, where this value must be smaller than a critical time step, which is determined by the mechanical properties of the contact law. In this method, increasing the time step value leads to deal with unbalanced forces, where the numerical resolution stability of the equations of motion is not ensured [4].

To face these issues, in this paper a viscoelastic contact law is developed and implemented in a particle-based code as an alternative to model asphalt mixtures dynamic properties. The
proposed contact model deals with larger time step values to reduce the computational time and assess the micromechanical properties of the model from macro-scale parameters at the contact particle. To validate the method and to determinate the contact model parameters, laboratory complex modulus tests in a two-point bending configuration were performed on trapezoidal samples.

The initial section (Sect. 2) of this paper set forth the experimental set-up and the determination of viscoelastic properties from laboratory tests. Then, the numerical procedures are outlined in Sect. 3, with the description of the numerical approach, the implementation of the contact model (Sect. 3.1) and the complex modulus test simulations (Sect. 3.2). Finally, Sect. 4 analyzes the results from numerical complex modulus test and compares with experimental data to validate the proposed contact model. A sensitivity analysis of model parameters is also performed in this section.

2. Laboratory work

This section presents experimental procedures for the complex modulus test with the evaluation of viscoelastic macroscopic parameters.

2.1. Complex modulus set-up

Complex modulus test were conducted in a two-point bending (2PB) configuration according to the EN 12697-26:2012 specification [13]. Four trapezoidal samples were prepared with Semi Coarse Asphalt Concrete (BBSG). Table 1 displays the particle size distribution (PSD) for this asphalt mixture. These aggregates come from the quarry of Bréfauchet in France and the bitumen is a 35/50 grade. The specimens were saw-cut from slabs prepared in laboratory, with the resulting dimensions: \( b = 25 \text{mm} \), \( B = 56 \text{mm} \), \( e = 25 \text{mm} \) and \( h = 250 \text{mm} \) (Fig. 1a). The mass of each trapezoidal sample \( M \) is on average 0.6kg and the mobile equipment mass at the head of the specimen \( m \) is 0.235kg.

For this series of trials the frequency values tested were set at 3, 6, 10, 25 and 40Hz while the selected temperatures were -10, 0, 10, 15, 20 and 30°C. For each couple temperature-frequency, a sinusoidal displacement \( z = A \sin(\omega t) \) is applied on the top of each sample. The amplitude \( A \) is set to \( 63 \times 10^{-6} \)m to obtain a strain \( \varepsilon \) less than \( 50 \times 10^{-6} \) to avoid fatigue damage. Each test is conducted for a total duration ranging from 30s to 120s. The peak values of force \( F_0 \), displacement \( z_0 \) and initial phase angle \( \Psi \) were measured during the last ten seconds of each test.

2.2. Determination of viscoelastic material properties

The viscoelastic behavior of an asphalt mixture under cyclic loading can be assessed by the norm of the complex modulus \( ||E^*|| \) and the associate phase angle \( \Phi \). To determinate these viscoelastic parameters, the real part \( E_1 \) and the imaginary part \( E_2 \) of the complex modulus must be calculated for each couple temperature-frequency. These values are provided by Eqs. 1 and 2 respectively [13]:

\[
E_1 = \gamma \left( \frac{F_0}{z_0} \cos(\Psi) + 10^{-6} \mu \omega^2 \right) \tag{1}
\]

\[
E_2 = \gamma \frac{F_0}{z_0} \sin(\Psi) \tag{2}
\]

\[
\gamma = \frac{12h^3}{e(B-b)^3} \left( 2 - b \frac{\mu}{2B} - \frac{3}{2} - \ln \frac{b}{B} \right) \tag{3}
\]

where \( \gamma \) (mm\(^{-1}\)) is the form factor depending on the geometrical values showed in Fig. 1a and \( \mu = 0.135M+m \) is the mass factor influencing the resultant force by inertial effects. Finally, the norm of the complex module and the associate phase angle are calculated using the following expressions:

\[
||E^*|| = \sqrt{E_1^2 + E_2^2} \tag{4}
\]
In this local frame ($\delta\vec{f}$) defined by the normal unit vector $\vec{n}$ with a viscoelastic model acting on distant contacts. For rigid particles to ensure the non-interpenetration criterion model mixes the original NSCD formulation [20, 21, 22, 23, 24] for rigid particles to ensure the non-interpenetration criterion with a viscoelastic model acting on distant contacts.

3. Numerical procedures

This section presents the steps to develop a contact model able to simulate a viscoelastic granular material based on the Non-Smooth Contact Dynamics (NSCD) method. Then, for the numerical modeling of the complex modulus test, the preparation protocol and the determination of model parameters are detailed.

3.1. Contact model description

In the aim to model an asphalt mixture during a complex modulus test, a viscoelastic contact model was developed and implemented in the LMGC90 software. The latter is capable of modeling a collection of rigid or deformable particles of various shapes by different algorithms [14, 15, 16, 17, 18, 19], (see https://git-xen.lmgc.univ-montp2.fr/ lmgc90/lmgc90_user/wikis/home). The created contact model mixes the original NSCD formulation [20, 21, 22, 23, 24] for rigid particles to ensure the non-interpenetration criterion with a viscoelastic model acting on distant contacts.

3.1.1. Non-smooth complementarity relations

For two particles candidate to the contact, it is possible to define a local frame at the particle-scale. The local frame is defined by the normal unit vector $\vec{n}$, which links the normal gap $\delta_n$ formed by the closer points between two particles. It is possible to associate to this normal vector two additional vectors of its tangential plane $\vec{t}$ and $\vec{s}$ to constitute an orthonormal base. In this local frame ($\vec{n}, \vec{t}, \vec{s}$), vector $\delta = (\delta_n, \delta_t, \delta_s)$ describes the normal and tangential gaps between two particles.

In the local frame, it is possible to describe the kinematic of rigid particles at contact as complementarity relations between velocities and contact forces. For a frictional contact, these relations can be expressed by the Signorini relation (Eq. 6) and the Coulomb friction law (Eq. 7), as following:

$$\begin{cases} 
\delta_n > 0 & \Rightarrow \quad R_n = 0 \\
\delta_n = 0 & \Rightarrow \quad U_n \geq 0, R_n \geq 0, U_n R_n = 0
\end{cases} \quad (6)$$

$$\begin{cases} 
\delta_t > 0 & \Rightarrow \quad R_t = 0 \\
U_t > 0 & \Rightarrow \quad R_t = -\mu R_n \\
U_t = 0 & \Rightarrow \quad -\mu R_n \leq R_t \leq \mu R_n \\
U_t < 0 & \Rightarrow \quad R_t = \mu R_n
\end{cases} \quad (7)$$

where $U_n$ and $U_t$ are the normal and tangential relative velocities, $R_n$ and $R_t$ are the normal and tangential local contact forces, and $\mu$ correspond to the coefficient of friction. This formulation for the Coulomb friction law is valid for both tangential directions $\vec{t}$ and $\vec{s}$. Equation 6 expresses mutual exclusion of particles at contact, where particle-interpenetration is not allowed. Figure 2 displays a graphical representation of these relations.

Besides the frictional interaction between rigid particles at contact, a viscoelastic behavior was included in the numerical modeling. For this purpose the Burger’s model was chosen. This rheological model is able to reproduce the viscoelastic behavior of asphalt mixtures [25, 26, 27]. It describes finely the creep, relaxation and dynamic properties of asphalt mixtures and has become a commonly used contact model in DEM simulation of these materials [9, 4, 5, 12, 6, 7]. The Burger’s model comprises a Maxwell model combined in series with a Kelvin-Voigt model. Here, $K_m$ is the Maxwell stiffness, $C_m$ is the Maxwell viscosity, $K_k$ is the Kelvin-Voigt stiffness and $C_k$ corresponds to the Kelvin-Voigt viscosity. Figure 3 shows the schema of the Burger’s model at the contact frame for the normal and tangential components. This viscoelastic model binds the particles whose gap distance $\delta_n$ is below the minimum diameter $d_{min}$ from the numerical PSD. This contact model is applied at the barycenter of each linked particle.

Assuming a perfectly isotropic behavior, the tangential mechanical properties of the model following both tangential directions $\vec{t}$ or $\vec{s}$ are set as equals. Thus, tangential stiffness and viscosities $K_{m_t}$, $C_{m_t}$, $K_{k_t}$ and $C_{k_t}$ are estimated as a function of the corresponding normal components as:
Normal components; (b) Tangential components.

Figure 3: Schema representation of the Burgers model in the local frame: (a) Normal components; (b) Tangential components.

\[ \delta_n < \delta_0 \Rightarrow U_n = 0, R_n = 0 \]
\[ \delta_n = \delta_0 \Rightarrow U_n \geq 0, R_n = 0 \]
\[ \delta_n > \delta_0 \Rightarrow U_n \geq 0, R_n = 0 \]

where \( \delta_0 \) is the initial normal gap \( \delta_n \) value at the initial equilibrium state, and \( R_n^\text{rit} \) represent a yield force. The \( R_n^\text{rit} \) value is computed as the product between the yield stress value and the projected cross-section of particles at contact. If one component of the local reaction \( R \) is larger than \( R_n^\text{rit} \) value, the bond between particles is broken and the frictional contact conditions (Eqs. 6 and 7) are applied instead. Both normal and tangential yield force values \( R_n^\text{rit} \) and \( R_t^\text{rit} \) are set as equals. The projected cross-section at contact is obtained by \( \pi r_{\text{min}}^2 \), where \( r_{\text{min}} \) is the minimum radius of two spheres at contact. For simplicity, the \( r_{\text{min}} \) value was considered for the cross-section calculation; nevertheless this approach is quite conservative. Other models could be used instead, such as calculating the effective radius between two particles or determining the cross-section based on models analogous to the capillary bonds [28, 29, 30].

In Fig. 4 is provided a representation of the relations between local velocities and contact forces for the viscoelastic interactions. Thus, for the developed contact model, the viscoelastic Burger’s model acts together with the frictional contact law. For the case of distant contacts \( (\delta_n > 0) \) the contact forces \( R \) are calculated using the Burger’s model. When a frictional contact is produced \( (\delta_n = 0) \) the generated contact forces are added to the resultant force for each contact.

### 3.1.2. Non-smooth dynamics

The NSCD formulation allows to describe the equations of motion for the frictional contact law in a compact form as:

\[ \ddot{U} = U^- + W \dot{R} \, dt \]  

where vectors \( \dot{U}^- \) and \( \ddot{U}^+ \) are the contact relative velocities at times \( t \) and \( t + dt \), respectively, and \( \dot{R} \) is the vector of local reaction at time \( t + dt \). The Delassus local matrix \( W \) contains the values of the inverse reduced inertia, where these components are calculated as a function of the contact geometry and inertia parameters of two particles at contact [24]. In this form, the accelerations are replaced by velocity jumps defined by \( \ddot{U}^- = \dot{U}^- \) and the equations of motion take the form of an equality between the change of momentum and the average reaction during the time step \( dt \).

To consider the viscoelastic behavior in this contact model, it is necessary to add the contribution of the Burger’s model into the general formulation of the NSCD method. In the Burger’s model, the gap between particles \( R \) is obtained by the sum of the local displacement in each section, i.e. the sum of Kelvin-Voigt section displacements \( \ddot{\delta} \) and the Maxwell section displacements \( \dot{\delta} \), and \( \ddot{\delta} \) due to stiffness and viscosity parts respectively, given by:

\[ \ddot{\delta} = \ddot{\delta}_k + \ddot{\delta}_m + \ddot{\delta}_c. \]
Considering an implicit time-integration scheme, the $\delta^+$ values are updated taking the values of the relative velocities $\tilde{U}^+$ at time $t + dt$ generated by the viscoelastic interaction, as:

$$\delta^+ = \delta^- + dt \tilde{U}^+$$  \hspace{1cm} (16)

where $\delta^-$ and $\delta^+$ correspond to the displacement at times $t$ and $t + dt$. Applying the formulation of Eq. 16 for each displacement term introduced in Eq. 15 allows to provide the complementary relative velocities for the Burger’s model, by:

$$\tilde{U}^+_k = \frac{\delta^+_k - \delta^-_k}{dt}$$  \hspace{1cm} (17)

$$\tilde{U}^+_{mk} = \frac{\delta^+_i - \delta^-_i}{dt}$$  \hspace{1cm} (18)

$$\tilde{U}^+_{mc} = \frac{\delta^+_i - \delta^-_i}{dt}$$  \hspace{1cm} (19)

where $\tilde{U}^+_k$, $\tilde{U}^+_{mk}$ and $\tilde{U}^+_{mc}$ correspond to the complementary velocities at the end of the time step generated by the Kelvin-Voigt section, the Maxwell stiffness and the Maxwell viscosity, respectively. Incorporating these complementary velocity terms into the general NSCD formulation (Eq. 14), leads to the following expression:

$$\tilde{U}^+ = \tilde{U}^- + \tilde{U}^+_k + \tilde{U}^+_{mk} + \tilde{U}^+_{mc} + \mathbf{W}\tilde{R}^+ dt$$  \hspace{1cm} (20)

Since the Burger’s model is in series, the local reaction vector at contact $\tilde{R}^+$ (normal and tangential components) at the end of a time step $dt$ can be expressed as:

$$\tilde{R}^+ = \tilde{R}^+_k = \tilde{R}^+_{mk} = \tilde{R}^+_{mc}$$  \hspace{1cm} (21)

where $\tilde{R}^+_k$, $\tilde{R}^+_{mk}$ and $\tilde{R}^+_{mc}$ correspond to the local reaction vectors at time $t + dt$ generated in the Kelvin-Voigt section, and in the Maxwell section by the stiffness and the viscosity respectively. Each local reaction can be obtained from Eqs. 22 through 24:

$$R^+_k = -K_k \delta^+_k - C_k U^+_k$$  \hspace{1cm} (22)

$$R^+_{mk} = -K_m \delta^+_i$$  \hspace{1cm} (23)

$$R^+_{mc} = -C_m U^+_i$$  \hspace{1cm} (24)

where $i$ stand for $n$, $t$ or $s$, while $K_k$, $C_k$, $K_m$ and $C_m$ provide the normal and tangential values of stiffness and viscosities for the Kelvin-Voigt and Maxwell sections respectively.

Finally, using Eqs. 22 to 24 into Eq. 20, together with the complementarity relations generated by the viscoelastic part from Eqs 12 and 13, provides the modified NSCD formulation, given by:

$$U^+_i + \frac{K_k \delta^+_i}{C_k} + \frac{R^+_i}{dt K_m} + \left[ \mathbf{W}U^+_i + \left( -\frac{1}{dt K_m} + \frac{1}{C_m} + \frac{1}{C_k} \right) \right] R^+_i dt = 0$$  \hspace{1cm} (25)

where $R^+_i$ is the local reaction vector at the beginning of the time step. To calculate all the velocities and contact forces in the system, at each time step, all kinematic constraints implied by frictional contacts and viscoelasticity from Eqs. 6, 7, 12 and 13, are considered together with Eqs. 14 and 25. The time-stepping schemes provided by Eqs. 14 and 25 are implicit but require explicit determination of the contact network at each time step.

This numerical problem is solved by means of an iterative procedure based on the nonlinear Gauss-Seidel method. The latter consists of solving a single contact problem with other contact forces being treated as known, and iteratively updating the reactions $\tilde{R}^+$ and velocities $\tilde{U}^+$ up to reach a convergence criterion. The iterations during a time step are stopped when the obtained $\tilde{R}^+$ values are stable regarding the update procedure.

For the developed contact model, the internal values are the initial normal gap $\delta_0$, the initial distance between barycenter of particles $l_0$, the projected cross-section of particles at contact and the Kelvin-Voigt displacement $\delta_k$. At the beginning
of the calculation process, the initial normal gap $\delta_n$ is assessed and stored as initial value $\delta_0$, together with the initial $l_0$ value, while the initial tangential gaps $\delta_t$ and $\delta_w$ are set to zero. According to Eq. 25, only the Kelvin-Voigt displacement $\delta_k$ is involved in the formulation among the displacement terms presented in Eq. 15. The normal and tangential components of $\delta_k$ are initially set to zero. Then, at the end of each time step, these components are updated using the Eq. 26 considering the $R^+$ current values.

$$\delta_k^+ = \frac{R^+_{k} \delta_k^- + C_k \delta_k^-}{K_k \delta_k^- + C_k}$$

At each time step, the yield stress criterion is checked. If the viscoelastic interaction breaks, the frictional contact law is applied instead. For our numerical simulations, a yield stress value of 2.7x10^6 Pa was fixed. This value is high enough to avoid fatigue damage during the test. This entire formulation allows implementing the developed viscoelastic contact model in the LMG90 software.

### 3.2. Complex modulus test modeling

#### 3.2.1. Sample preparation

The numerical samples are composed of spherical rigid particles, where each particle has a diameter ranging from 2mm to 10mm (see Tab. 2), based on the experimental PSD displayed in Tab. 1. Here, the graded curve was cut at 2mm to avoid modeling all the fines, which are considered as a part of the mastic, in the aim to reduce the number of particles in the system and consequently the computational time. Initially, for each particle-size, a specific number of spherical particles are created according to the particle size distribution. The generated spheres are disposed randomly within a cubic lattice with 0.25m length, 0.025m wide and 1.6m height. In this cubic lattice the length of each lattice element is 10mm to fit the largest particles. The initial bulk density for all particles is 2600 kg.m$^{-3}$. The preparation protocol consists in first pouring the initial spherical particles by gravity into a box composed of six plates that fit the cubic lattice described above (Fig. 5a). During this stage, the Coulomb friction law is applied for both particle-particle and particle-wall contacts, where the coefficient of friction is set to 0.2 for every contact. At the end of the deposition process, no granular segregation phenomenon was noticed (see Tab. 2).

The macroscopic model parameters allow to determinate the complex modulus from the resulting strain and stress as [31, 32, 5]:

$$||D'|| = \sqrt{D''(\omega)^2 + D'''(\omega)^2}$$

Both the real and the imaginary part of the complex compliance, $D'$ and $D''$, respectively, can be obtained by:

$$D'(\omega) = \frac{1}{E_m} + \frac{E_k}{E_k^2 + \omega^2 \eta_k^2}$$

$$D''(\omega) = \frac{\omega \eta_k}{E_m} + \frac{\omega \eta_k}{E_k^2 + \omega^2 \eta_k^2}$$

where $E_m$, $\eta_m$, $E_k$ and $\eta_k$ correspond to the macroscopic Maxwell and Kelvin-Voigt stiffness and viscosities respectively. The macroscopic model parameters allow to determinate the norm of the complex modulus as $||E''|| = ||D'||^{-1}$ and the phase angle by $\Phi = \arctan(D''/D')$. Using the established analytical expressions for $||E''||$ and $\Phi$ the best fit macroscopic parameters regarding experimental data can be estimated. The curve.
Figure 5: Snapshots for several stages during the preparation process. (a) Initial deposit by gravity. (b) Numerical sample before compaction. (c) Numerical sample at the end of the preparation.

Table 3: Burger’s model parameters used in numerical simulations.

<table>
<thead>
<tr>
<th>T (°C)</th>
<th>$E_m$ (Pa)</th>
<th>$\eta_m$ (Pa.s)</th>
<th>$E_k$ (Pa)</th>
<th>$\eta_k$ (Pa.s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>$3.98\times10^7$</td>
<td>$6.28\times10^7$</td>
<td>$9.56\times10^6$</td>
<td>$1.49\times10^7$</td>
</tr>
<tr>
<td>0</td>
<td>$3.55\times10^7$</td>
<td>$2.26\times10^7$</td>
<td>$2.95\times10^6$</td>
<td>$3.46\times10^6$</td>
</tr>
<tr>
<td>10</td>
<td>$2.78\times10^7$</td>
<td>$6.26\times10^6$</td>
<td>$8.46\times10^5$</td>
<td>$9.94\times10^5$</td>
</tr>
<tr>
<td>15</td>
<td>$2.43\times10^7$</td>
<td>$3.71\times10^6$</td>
<td>$4.84\times10^5$</td>
<td>$7.73\times10^5$</td>
</tr>
<tr>
<td>20</td>
<td>$2.02\times10^7$</td>
<td>$1.79\times10^6$</td>
<td>$2.72\times10^5$</td>
<td>$4.61\times10^5$</td>
</tr>
<tr>
<td>30</td>
<td>$1.30\times10^7$</td>
<td>$3.45\times10^5$</td>
<td>$9.31\times10^4$</td>
<td>$1.31\times10^5$</td>
</tr>
</tbody>
</table>

fitting was performed by means of an optimization algorithm from Python programming language, for each temperature. Finally, the microscopic normal model parameters ($K_{m}, C_{m}$, $K_{k}$, and $C_{k}$) are obtained from the macroscopic model parameters ($E_{m}, \eta_{m}, E_{k}$, and $\eta_{k}$) through:

$$K_{mn} = \frac{\pi r_{min}^2}{l_0} E_{mn}$$

(30)

$$C_{mn} = \frac{\pi r_{min}^2}{l_0} \eta_{mn}$$

(31)

$$K_{kn} = \frac{\pi r_{min}^2}{l_0} E_{kn}$$

(32)

$$C_{kn} = \frac{\pi r_{min}^2}{l_0} \eta_{kn}$$

(33)

where $r_{min}$ correspond to the minimum radius of two particles in contact and $l_0$ is the initial distance between the barycenters of two particles (see Fig. 3). Then, tangential components can be assessed from normal ones using Eq. 8 through Eq. 11. This approach creates a number of parameters at the micro-scale equal to the number of particles in contact within the sample. Table 3 displays the set of calibrated parameters determined with this procedure for each temperature.

To ensure numerical stability in the resolution, the time step length must be smaller than the characteristic time scale of the system. First, it is possible to quantify the typical time of rearrangement of grains $d_t = \sqrt{m_g/(pd_g)}$, with the average values of particle mass $m_g \approx 2.86\times10^{-3}$kg, pressure $p \approx 2.79\times10^3$Pa, and particle size $d_g \approx 2.52\times10^{-3}$m. In our system, the $d_t$ value is about $2\times10^{-3}$s. On the other hand, due to the elasticity in the proposed contact model, a second time scale can be identify as $d_t = \sqrt{m_g/K_{eq}}$, where $K_{eq}$ is the equivalent contact stiffness. In the proposed model, at the beginning of the calculation time, $K_{eq} \approx K_m$. For the case of 15°C, a $d_t$ value of $2\times10^{-3}$s is obtained.

The influence of the time step $dt$ on the total reaction measured on the top plate was assessed. Figure 6 displays the curves for the total reaction as a function of time for several values of $dt$ varying from $2\times10^{-5}$s to $2\times10^{-4}$s. These results show that force values can reach a stable measure with a time step value of $5\times10^{-5}$s, even if the latter is larger than the identified time scale.
The time step was set to $5 \times 10^{-5}$ s in all simulations and the number of time steps needed to perform five loading cycles ranging from $3.33 \times 10^4$ to $2.5 \times 10^3$ for 3 Hz and 40 Hz respectively. Using this time step value implies that the total computational time in these numerical simulations is divided by 2.5, compared to the typical time scale used in classic DEM approaches. The CPU time was $2 \times 10^{-4}$ s per particle and per time step on a Dell computer of speed 2.1 GHz.

4. Validation of the numerical modeling

4.1. Comparison between experimental and numerical data

The numerical complex modulus tests were conducted by subjecting the top plate to the same sinusoidal displacement as experimental tests. Then, five cycles were applied for each sample, measuring the displacement and the total reaction on the top plate as a function of time. The obtained force and displacement curves allow to determine the peaks values $F_0$ and $z_0$ and calculate $\|E^*\|$ and $\Phi$ from numerical trials. Figure 7 displays an example of the imposed displacement and the corresponding force measured at the top plate. As expected, the resultant curve of the total reaction follows the sinusoidal wave imposed by the displacement. The phase angle between these two curves allows to calculate $\Phi$. For all samples, six temperatures and five frequencies were tested such as for experimental trials.

Figure 8 depicts different stages during the numerical test. The mean velocity of particles increases as a function of the height, from zero for the particles bonded to the bottom plate, up to a maximum value for the particles in contact with the top plate. The numerical sample follows the movement imposed by the top plate as an viscoelastic beam without a local effect generated by the loading plate.

To validate the numerical model, Fig. 9 and 10 display the experimental and numerical results for the norm of complex modulus and phase angle isotherms respectively. One can note that the average numerical values from the simulations are just in a fairly good agreement with the values for $\|E^*\|$ found through experiments for all the tested frequencies. On the other hand, the numerical results for the phase angle $\Phi$ are just in a fairly good agreement with the experimental results. These isotherms show some fluctuations around the average experimental data.

In the aim to assess the forecasting error, Tab. 4 gives the results of the mean relative error (MRE) between experimental and numerical data for $\|E^*\|$ and $\Phi$ for each temperature. As we can see in Fig. 9, the numerical values of $\|E^*\|$ are very close to experimental ones, with a MRE smaller than 4% for all temperatures. On the other hand, the MRE values for $\Phi$ remain below 10% except for the case of $-10^\circ$C. These variations were already pointed out in Fig. 10. This high variability at low temperature is produced in part for a ratio of two small numbers in the calculation of MRE, both the numerical and experimental $\Phi$ values. Despite the variations in the prediction for some temperatures...
regarding $\Phi$ values, the general results can be considered as encouraging for a validation of the proposed contact model for the study of asphalt mixtures.

### 4.2. Sensitivity analysis of model parameters

To complete the study and to take into account a lack of accuracy into the calibration of Burger’s model parameters, Figs. 11 to 14 display the influence of these parameters in the determination of $|E^*|$ and $\Phi$. This sensitivity analysis was performed for 20°C, where each model parameter was fixed at ±50% regarding the average values given in Tab. 3.

Regarding the effect of the model parameters variation on $|E^*|$ values, $E_m$ shows the most important influence compared to the other parameters. Theoretically, the stiffness of the Maxwell model can be express as $E_m = \lim_{f \to \infty} |E^*|$. Hence, the influence of $E_m$ values on $|E^*|$ increase as the frequency increases, as displayed in Fig. 11. A similar trend but with a lower magnitude can be highlighted for the Kelvin-Voigt viscosity $\eta_m$. Concerning the influence on the phase angle, a variation of 50% on each parameter of the contact model, leads to significant fluctuations in the obtained $\Phi$ values. It seems difficult to figure out the origin of this high sensibility regarding the phase angle. One can point out the need of a correct choice of the model parameters to avoid biases in the prediction of this material property.

### 5. Concluding remarks

In this paper NSCD simulations of the complex modulus test in a 2PB configuration were performed. To model the viscoelastic phase surrounding rigid spherical particles in an asphalt mixture, a viscoelastic contact law based on the Burger’s model was implemented in the LMGC90 software. These simulations reproduce the experimental campaign, where four trapezoidal samples were subjected to five frequencies and six temperatures.

The outcomes in this study show that the implemented viscoelastic contact law based on the Burger’s model is able to reproduce the complex modulus properties of asphalt mixtures.
For different analysis conducted in this paper, the numerical results of these simulations were in a good agreement regarding experimental data. For isotherm curves, experimental and numerical data of the norm of the complex modulus $|E^*|$ collapse in the same curve. The numerical phase angle $\Phi$ values follow the trend of experimental ones, despite some fluctuations around the average values.

Sensitivity analysis shows that the Maxwell stiffness has an important influence in the calculation of $|E^*|$, increasing with the frequency values. For the case of $\Phi$, the contact model is quite sensitive to changes in parameter values, diverging from the average tendency.

In conclusion, this work can be considered as encouraging for the validation of the proposed contact model for the study of asphalt mixtures features such as creep, rutting and fatigue damage. In this study, the parameters of the Burger’s model were calibrated using analytical expressions to assess the micromechanical properties for the contact model from macro-scale parameters obtained by experimental tests.

In this preliminary work, the numerical test was modeled using spherical aggregates. In a granular material, as an asphalt mixture, the particle shape plays a major role regarding fabric anisotropy, force transmission and friction mobilization. This aspect must be taken into account when extrapolating these results to actual systems.

Further works will focus on the characterization of the rheological properties of mastic to incorporate them directly into the contact model. This feature allows forecasting the viscoelastic properties of an asphalt mixture from its formulation. The next steps in this work will be to study the mechanical behavior of multi-layer road structures and the influence of particle shape and different particle size distributions on the mechanical performances taking into account actual aggregates from 3D scan to obtain more realistic asphalt mixtures. The numerical results showed in this study were obtained for small amplitude vibrations. In the case of large deformation or failure analysis, the yield stress value must be calibrated to extend the developed model to study cases such as crack propagation or fatigue damage.

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